Transformations of Graphs

Transformations
INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to transform graphs. It will help you to identify and support students who have difficulty:

- Describing transformations that carry a figure onto itself.
- Rotating, reflecting, or translating a figure in the coordinate plane.

GEORGIA STANDARDS OF EXCELLENCE

This lesson involves mathematical content in the standards from across the grades, with emphasis on:

- MGSE.9-12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

- MGSE.9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Standards for Mathematical Practice

- SMP1 - Make sense of problems and persevere in solving them.
- SMP3 - Construct viable arguments and critique the reasoning of others.
- SMP4 - Model with mathematics.
- SMP5 – Use appropriate tools strategically.

INTRODUCTION

This lesson is structured in the following way:

Before the Lesson,

- Students work individually on an assessment task designed to reveal current understandings and difficulties. You then review their work and create questions to improve their understanding.

At the Start of the Lesson,

- Display reproductions of art by M.C. Escher. Have students look for transformations. Discuss.

During the Lesson,

- During Session One, students will manipulate polygons in order to identify all transformations that carry a figure onto itself.
- During Session Two, students will graph multiple transformations for a given figure. Group members will work together to determine which given solution is correct.

After the Whole-Group Class Discussion,

- Student pairs will explain why each choice is correct or what misconception causes it to be incorrect. Student volunteers will present while the teacher “scribes” the problems.
- Finally, students complete the post-assessment to demonstrate what they have learned.
MATERIALS REQUIRED

Each individual student will need: Pre-assessment: *I Need a Drink*, Post-assessment: *Crack the Safe*, pencil, eraser, white boards, and dry erase marker.

Each small group will need: polygon sheet, 360° protractor, ruler, scissors, activity recording sheet, graphing problems, answer choices for each graphing problem, and tape.

TEACHER PREP REQUIRED

The only teacher prep is copying and cutting the Protractor Sheet into two pieces.

TIME NEEDED:

For Pre-Assessment: 15 minutes  
For Lesson: 95 minutes  
For Post: 15 minutes  
Other:

Special Notes about timing: Approximately 15 minutes will be used before the lesson for pre-assessment. Day 1 requires approximately 15 minutes whole class introduction and approximately 30 minutes for the activity lesson. Day 2 requires approximately 30 minutes for the activity lesson, 20 minutes whole class plenary discussion, and a 15 minute post assessment. Timings are approximate and will depend on the needs of the class.

FRAMING FOR THE TEACHER:

This formative assessment lesson will show if students truly understand transforming geometric figures both in and out of a coordinate plane. Students must identify transformations that carry a figure onto itself. Students must be able to perform or identify multiple transformations that result in a new figure. The state identified this skill as a weakness after the 2013 Coordinate Algebra EOCT results were analyzed.

FRAMING FOR THE KIDS:

Say to the students:

*This activity will take about 2 1/2 days for us to complete.*

*The reason we are doing this is to be sure that you understand transformations before we move on to a new idea. You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!*
PRE-ASSESSMENT BEFORE THE LESSON

ASSESSMENT TASK:
Name of Assessment Task: I Need a Drink
Time This Should Take: (15 minutes)

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of I Need a Drink.

Briefly introduce the task and help the class to understand the problem and its context.

*Spend 15 minutes working individually on this task. Read through the task and try to answer it as carefully as you can. Show all your work so that I can understand your reasoning. Don’t worry if you can’t complete everything. There will be a lesson that should help you understand these concepts better. Your goal is to be able to confidently answer questions similar to these by the end of the next lesson.*

Students should do their best to answer these questions, without teacher assistance. It is important that students are allowed to answer the questions on their own so that the results show what students truly do not understand.

Students should not worry too much if they cannot understand or do everything on the pre-assessment, because in the next lesson they will engage in a task which is designed to help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently.

This is their goal.
COLLABORATION TIME/READING STUDENTS RESPONSES

You Will Not “Grade” These!

Collect student responses to the task. It is helpful to read students’ responses with colleagues who are also analyzing student work. Make notes (on your own paper, not on their pre-assessment) about what their work reveals about their current levels of understanding, and their approaches to the task. You will find that the misconceptions reveal themselves and often take similar paths from one student to another, and even from one teacher to another. Some misconceptions seem to arise very organically in students’ thinking. Pair students in the same classes with other students who have similar misconceptions. This will help you to address the issues in fewer steps, since they’ll be together. (Note: Pairs are better than larger groups for FAL’s because both must participate in order to discuss!)

You will begin to construct Socrates-style questions to try and elicit understanding from students. We suggest you write a list of your own questions; however some guiding questions and prompts are also listed below as a jumping-off point.

GUIDING QUESTIONS

COMMON ISSUES | SUGGESTED QUESTIONS AND PROMPTS
--- | ---
Student has a hard time getting started. | • Can you list the three transformations we have studied and think about what they are?
Student incorrectly identifies the data from the graph. For example, student reverses x and y coordinates. | • Can you explain what the ordered pair (2,3) means?  
• Think of this point (3,5). Give me “driving” instructions from the origin.
Student uses the wrong axis when reflecting. | • Can you point out the x-axis? (or y-axis)  
• Do you know any “tricks” to remember the x-axis and y-axis?
Student has difficulty using the algebraic expressions to make the transformation. | • What are you doing to each x-coordinate?  
• What are you doing to each y-coordinate?
Student has difficulty understanding Question 1. | • Can you circle the words that tell you what to look for in this problem?  
• How can you make the polygon match the hole?  
• Are there any other times it would match?
Student adds little or no explanations as to why answers are formed. | • How would you explain this to someone unfamiliar with these questions?  
• What math can you do to justify your answer?
LESSON DAY
SUGGESTED LESSON OUTLINE:

Part 1: Whole-Class Introduction:  

Time to Allot: (15 minutes)

- Show the class the slide “Math in Escher’s Art – One”.
- Using white boards, have students write down any transformation that they see. Call on students to share and explain by pointing out the transformation. Most students will see translations in this piece of art. Review isometry and have students explain why it is important in this piece of art.
- Show the class the slide “Math in Escher’s Art – Two”.
- Repeat white board response and discussion. Most students will see rotations in this piece of art. Discuss degree of rotation as well as point of rotation. Review isometry and have students explain why it is important in this piece of art.
- Show the class the slide “Math in Escher’s Art-Three”.
- Repeat white board response and discussion. Most students will see rotations in this piece of art. Discuss degree of rotation and point of rotation. Students should also see reflection in this artwork. Discuss where the line of reflection would be for both the angel and the devil figures. Review isometry and have students explain why it is important in this piece of art.
Part 2: Collaborative Activity: Time to Allot: (60 minutes)

Collaborative activity: Carry Onto Itself (Approximately 30 minutes)

- Put students into their pairs according to your analysis of student errors from the pre-assessment.
- Distribute polygon sheets, 360° protractor sheet, and recording sheet. Have students carefully cut out polygons and protractor.
- Each group should start with the triangle and use the protractor to determine the degree of rotation needed to make the figure look exactly the same as it was in the starting position. Challenge student to rotate both clockwise and counter-clockwise. Record answers on the recording sheet.
- Each group should use the ruler to create lines of symmetry. Check the lines by folding to see if the two parts match. Record answers on the recording sheet.
- Each group will repeat the process with the additional polygons. Groups should complete the summary questions for each section.
- Teacher circulation and monitoring of the summary questions is critical.

Collaborative activity: Graphing Transformations (Approximately 30 minutes)

- Groups re-assemble and retrieve their materials from Day 1.
- Pass out two graphing problems. Students work in pairs to make transformations in the order described. Transformed points should be recorded.
- Once both graphs are completed, each group will receive multiple choice answers for each problem. The group should determine the correct answer and tape their graph under the appropriate solution on the wall. (Teachers: Cut apart one multiple choice sheet and tape answer choices to the wall about 2 feet apart.)

During both Collaborative Activities, the Teacher has 3 tasks:

- Circulate to students’ whose errors you noted from the pre-assessment and support their reasoning with your guiding questions.
- Circulate to other students also to support their reason in the same way.
- Make a note of student approaches for the summary (plenary discussion). Some students have interesting and novel solutions!

Part 3: Plenary (Summary) Discussion: Time to Allot: (20 minutes)

- For each answer choice posted on the wall, have a pair of students explain why the group chose that answer. Remember to question why the mistake was made not just change incorrect answers to correct answers.
- The teacher should scribe (or script) the key points and recognize specific students.

NOTE: “Scribing” helps to increase student buy-in and participation. When a student answers your question, write the student’s name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!
Part 4: Improving Solutions to the Assessment Task    Time to Allot: (15 minutes)

The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment. In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no “X” mark means that it must have been right. Until students become accustomed to UNGRADED FORMATIVE assessments, they may naturally do this. Teachers often report success by displaying a list of the guiding questions to keep in mind while they improve their solutions.

- Return student pre-assessments.
- If you did not mark pre-assessments with questions, display a list of questions on the board.
- Distribute post-assessments and have student spend approximately 15 minutes completing.
  Note: If you are running short of time, this post-assessment can be done the following day.
**PRE- ASSESSMENT (Answer Key)**

**Problem 1:** It is summer break and your mom has had enough! You and your friends are constantly opening the refrigerator to get drinks. Your mom installed a lock on the door. The lock has a spinning pentagon with a sensor that causes the refrigerator door to unlock for one minute each time it matches exactly with the pentagon below it. The pentagon rotates 1 degree every minute. How often can you open the door to get a drink?

*Show your answer and work below. Be sure to explain how you decided your answer.*

The door will unlock when it is 72, 144, 216, 288, and 360 (same as 0) minutes.

\[
\frac{360}{5} = 72
\]

Since all the way around is a complete circle, the total degrees is 360. There are 5 congruent sides so there are 5 congruent angles. Divide the 360 by 5 to get 72. Since it is a regular polygon, every 72 minutes it matches. That means it will open at 72 min, 144 min, 216 min, 288 min, 360 min, etc.

**Problem 2:** Graph the image of quadrilateral ABCD after the following transformations. Record your coordinates in the chart after each transformation.

Transformation 1: Reflection across the y-axis.

Transformation 2: \((x, y) \rightarrow (x + 8, y - 2)\)

<table>
<thead>
<tr>
<th>Pre-Image</th>
<th>Transformation #1</th>
<th>Transformation #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (2,4)</td>
<td>A’ (-2,4)</td>
<td>A’’ (6,2)</td>
</tr>
<tr>
<td>B (4,7)</td>
<td>B’ (-4,7)</td>
<td>B’’ (4,5)</td>
</tr>
<tr>
<td>C (6,5)</td>
<td>C’ (-6,5)</td>
<td>C’’ (2,3)</td>
</tr>
<tr>
<td>D (4,2)</td>
<td>D’ (-4,2)</td>
<td>D’’ (4,0)</td>
</tr>
</tbody>
</table>

If you reversed the order of the transformations, would the final image be the same? Explain.

*No, Point A is (2,4). If we did transformation 2 first A’’(10,2), then transformation 2 would make A’’’(-10,2) instead of (6,2).*
The lock shown below has been installed on the safe to your office. To open the safe, you must understand how a shape maps onto itself. The “key” to the safe has two polygons that rotate together at a constant rate of 1 degree per second. To open the safe, you must enter the time that the polygons rotate. The safe will only open if the polygons match the lock exactly, but are not in the “home” position. Only one attempt can be made each 24 hours. Good Luck! Show your answer and work below. Be sure to explain how you decided your answer.

The door will unlock when it is 180.

\[
\frac{360}{4} = 90 \quad \frac{360}{6} = 60
\]

The lock will open when both polygons match the original.

The multiples of 90 are 90, 180, 270, 360.

The multiples of 60 are 60, 120, 180, 240, 300, and 360.

The shapes both match at 180 seconds.

Problem 2: Graph the image of quadrilateral ABCD after the following transformations. Record your coordinates in the chart after each transformation.

Transformation 1: Reflection across the x-axis.
Transformation 2: \((x, y) \rightarrow (x - 9, y + 3)\)

If you reversed the order of the transformations, would the final image be the same? Explain.

No, Point A is (2,4). If we did transformation 2 first A'(-7,7), then transformation 2 would make A''(-7,-7) instead of (-7,-1).

Activity Recording Sheet Answer Key

Part 1: Cut out the polygons. Divide the polygons into two groups – regular polygons and non-regular polygons.
You will work with the regular polygons first. You are looking for all possible rotations of each polygon that result in the polygon carrying onto itself (looks just like it did before you transformed it). Use the protractor to determine the degree of rotation. Record your answers in the chart provided.

<table>
<thead>
<tr>
<th>Name of Polygon</th>
<th>Degrees of Rotation when polygon will carry onto itself.</th>
<th>Reflection Lines when polygon will carry onto itself.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td>360/3 = 120, 120°, 240°, and 360°</td>
<td>3 lines; each splits a side in half and connects to opposite vertex</td>
</tr>
<tr>
<td>Square</td>
<td>360/4 = 90, 90°, 180°, 270°, and 360°</td>
<td>4 lines; diagonals, horizontal, and vertical</td>
</tr>
<tr>
<td>Regular pentagon</td>
<td>360/5 = 72, 72°, 144°, 216°, 288°, and 360°</td>
<td>5 lines; each splits a side in half and connects to the opposite vertex</td>
</tr>
<tr>
<td>Regular hexagon</td>
<td>360/6 = 60, 60°, 120°, 180°, 240°, 300°, and 360°</td>
<td>6 lines; each either splits parallel sides in half or splits opposite angles in half</td>
</tr>
</tbody>
</table>

**Part 2**: Now use your ruler to draw in any lines of symmetry (or reflection lines). Test to make sure that the figure will carry onto itself by folding the polygon along the line. If the two halves do not match, it is not a reflection line. Record your answers in the chart provided.

**Part 3**: Explain any patterns that you see in the degrees of rotation and in the reflection lines.  
*When you have regular polygons, the number of lines of symmetry (or reflection lines) is always the same as the number of sides. Odd numbers split a side and the opposite vertex whereas even number of sides split parallel sides or opposite angles. The degree of rotation is always 360/n.*

**Part 4**: Now use your non-regular polygons to repeat both Part 1 and Part 2. Record your answers in the chart provided.

<table>
<thead>
<tr>
<th>Name of Polygon</th>
<th>Degrees of Rotation when polygon will carry onto itself.</th>
<th>Reflection Lines when polygon will carry onto itself.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trapezoid or Isosceles Trapezoid</td>
<td>none</td>
<td>1 lines; Vertical right in middle of the top and bottom bases</td>
</tr>
<tr>
<td>Rectangle</td>
<td>180°; it only works flipped upside down</td>
<td>2 lines; Horizontal and vertical</td>
</tr>
</tbody>
</table>

**Part 5**: Explain any patterns that you see in the degrees of rotation and in the reflection lines.  
*A figure must have rotational symmetry to have a degree of rotation.*

**Answers to graphing:**
PRE-ASSESSMENT – I Need a Drink!

Problem 1: It is summer break and your mom has had enough! You and your friends are constantly opening the refrigerator to get drinks. Your mom installed a lock on the door. The lock has a spinning pentagon with a sensor that causes the refrigerator door to unlock for one minute each time it matches exactly with the pentagon below it. The pentagon rotates 1 degree every minute. How often can you open the door to get a drink? Show your answer and work below. Be sure to explain how you decided your answer.

Problem 2: Graph the image of quadrilateral ABCD after the following transformations. Record your coordinates in the chart after each transformation.

Transformation 1: Reflection across the y-axis.
Transformation 2: \((x, y) \rightarrow (x + 8, y - 2)\)

If you reversed the order of the transformations, would the final image be the same? Explain.

<table>
<thead>
<tr>
<th>Pre-Image</th>
<th>Transformation #1</th>
<th>Transformation #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (2,4)</td>
<td>A'</td>
<td>A''</td>
</tr>
<tr>
<td>B (4,7)</td>
<td>B'</td>
<td>B''</td>
</tr>
<tr>
<td>C (6,5)</td>
<td>C'</td>
<td>C''</td>
</tr>
<tr>
<td>D (4,2)</td>
<td>D'</td>
<td>D''</td>
</tr>
</tbody>
</table>
POST-ASSESSMENT – Crack the Safe!

The lock shown below has been installed on the safe to your office. To open the safe, you must understand how a shape maps onto itself. The “key” to the safe has two polygons that rotate together at a constant rate of 1 degree per second. To open the safe, you must enter the time that the polygons rotate. The safe will only open if the polygons match the lock exactly, but are not in the “home” position. Only one attempt can be made each 24 hours. Good Luck!  

Show your answer and work below. Be sure to explain how you decided your answer.

Problem 2: Graph the image of quadrilateral ABCD after the following transformations. Record your coordinates in the chart after each transformation.

![Diagram of quadrilateral ABCD and its transformations]

Transformation 1: Reflection across the x-axis.
Transformation 2: \((x, y) \rightarrow (x - 9, y + 3)\)

<table>
<thead>
<tr>
<th>Pre-Image</th>
<th>Transformation #1</th>
<th>Transformation #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (2,4)</td>
<td>A'</td>
<td>A''</td>
</tr>
<tr>
<td>B (4,7)</td>
<td>B'</td>
<td>B''</td>
</tr>
<tr>
<td>C (6,5)</td>
<td>C'</td>
<td>C''</td>
</tr>
<tr>
<td>D (4,2)</td>
<td>D'</td>
<td>D''</td>
</tr>
</tbody>
</table>

If you reversed the order of the transformations, would the final image be the same? Explain.
COLLABORATIVE ACTIVITY

Day 1 Activity: Master for Polygons
Activity Recording Sheet: Carry Onto Itself
**Part 1:** Cut out the polygons. Divide the polygons into two groups – regular polygons and non-regular polygons. You will work with the regular polygons first. You are looking for all possible rotations of each polygon that result in the polygon carrying onto itself (looks just like it did before you transformed it). Use the protractor to determine the degree of rotation. Record your answers in the chart provided.

<table>
<thead>
<tr>
<th>Name of Polygon</th>
<th>Degrees of Rotation when polygon will carry onto itself.</th>
<th>Reflection Lines when polygon will carry onto itself.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Part 2:** Now use your ruler to draw in any lines of symmetry (or reflection lines). Test to make sure that the figure will carry onto itself by folding the polygon along the line. If the two halves do not match, it is not a reflection line. Record your answers in the chart provided.

**Part 3:** Explain any patterns that you see in the degrees of rotation and in the reflection lines.

**Part 4:** Now use your non-regular polygons to repeat both Part 1 and Part 2. Record your answers in the chart provided.

<table>
<thead>
<tr>
<th>Name of Polygon</th>
<th>Degrees of Rotation when polygon will carry onto itself.</th>
<th>Reflection Lines when polygon will carry onto itself.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Part 5:** Explain any patterns that you see in the degrees of rotation and in the reflection lines.

**Day 2 Activity: Graphing Multiple Transformations – Problem 1**

Graph the image of quadrilateral FGHI after the following transformations. Record your coordinates in the chart after
Transformation 1: Translation \((x,y) \rightarrow (x+1, y-1)\)
Transformation 2: Reflection across the line \(x = -3\)

<table>
<thead>
<tr>
<th>Pre-Image</th>
<th>After Transformation #1</th>
<th>After Transformation #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (-2,14)</td>
<td>F'</td>
<td>F''</td>
</tr>
<tr>
<td>G (-8,8)</td>
<td>G'</td>
<td>G''</td>
</tr>
<tr>
<td>H (-2,2)</td>
<td>H'</td>
<td>H''</td>
</tr>
<tr>
<td>I (4,8)</td>
<td>I'</td>
<td>I''</td>
</tr>
</tbody>
</table>

Day 2 Activity: Graphing Multiple Transformations – Problem 2
Graph the image of triangle WUV after the following transformations. Record your coordinates in the chart after each transformation.
Transformation 1: Translation \((x, y) \rightarrow (x+11, y-15)\)
Transformation 2: Rotation \(270^\circ\) clockwise around the origin.

Day 2 Activity: Graphing Multiple Transformations

Below you will find 3 answer choices for each problem. Find the choice that matches your answer. Tape your work on the wall below the answer choice that you agree with. Remember to put your name on your work!

Problem 1:

<table>
<thead>
<tr>
<th>Pre-Image</th>
<th>After Transformation #1</th>
<th>After Transformation #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W (-3,8))</td>
<td>(W')</td>
<td>(W'')</td>
</tr>
<tr>
<td>(U (-11,8))</td>
<td>(U')</td>
<td>(U'')</td>
</tr>
<tr>
<td>(V (-7,4))</td>
<td>(V')</td>
<td>(V'')</td>
</tr>
</tbody>
</table>
Problem 2: