Algebra2/Trig Chapter 9 Packet

In this unit, students will be able to:

- Use the Pythagorean theorem to determine missing sides of right triangles
- Learn the definitions of the sine, cosine, and tangent ratios of a right triangle
- Set up proportions using sin, cos, tan to determine missing sides of right triangles
- Use inverse trig functions to determine missing angles of a right triangle
- Solve word problems involving right triangles
- Identify and name angles as rotations on the coordinate plane
- Determine the sign (+/-) of trig functions on the coordinate plane
- Determine sin, cos, and tangent of “special angles” (exact trig values)
- Determine reference angles for angles on the coordinate plane
- Determine the sine, cosine, and tangent of angles on the coordinate plane
- Do all of the above, using the reciprocal trig functions

Name:____________________________________

Teacher:____________________________________

Pd: ______
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Day 1: Right Triangle Trigonometry

Trigonometry means “triangle measure”. From now on, the vertices of a right triangle will always be named with a capital letter. The side opposite each vertex will have the same letter, but lower-case.

The Pythagorean theorem is an equation that relates the three sides of a right triangle.

The trigonometric (trig) functions are equations that relate two sides of a triangle with an angle of the triangle.

The three trig functions we will start with are sine (sin), cosine (cos), and tangent (tan.)

<table>
<thead>
<tr>
<th>The Sine Ratio</th>
<th>The Cosine Ratio</th>
<th>The Tangent Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ratio between the leg opposite a given angle of a right triangle and the triangle’s hypotenuse.</td>
<td>The ratio between the leg adjacent to a given angle of a right triangle and the triangle’s hypotenuse.</td>
<td>The ratio between the leg opposite a given angle of a right triangle and the leg adjacent to the same angle.</td>
</tr>
<tr>
<td>( \sin \theta = \frac{\text{leg opposite } \theta}{\text{hypotenuse}} )</td>
<td>( \cos \theta = \frac{\text{leg adjacent to } \theta}{\text{hypotenuse}} )</td>
<td>( \tan \theta = \frac{\text{leg opposite } \theta}{\text{leg adjacent to } \theta} )</td>
</tr>
</tbody>
</table>

An easy way to remember the above functions is to use the mnemonic: “SOH CAH TOA”. Remember, the hypotenuse is never used as an opposite or adjacent side!
Concept #1: Writing the “3” Trig Ratios

Example 1: Determine the trig ratios for the triangle below.

You Try It!

When determining trig ratios, you might need to find the missing side of a triangle using the Pythagorean’s Theorem from your knowledge of geometry.

Find:

PR:

\( \sin \angle P = \)

\( \cos \angle P = \)

\( \tan \angle P = \)

\( \sin \angle R = \)

\( \cos \angle R = \)

\( \tan \angle R = \)

Evaluate with a calculator (make sure you are in degrees mode). Round answers to nearest hundredth.

a) \( \sin 48^\circ = \)

b) \( \tan 45^\circ = \)

c) \( \cos 72^\circ = \)
Concept 2: Using Trig Functions to find Missing Sides of Right Triangles

Because the trigonometric ratios stay constant, if you are given a right triangle with one acute angle and any of the sides, you can use the trig ratios to find another missing side.

**Example:** Find the missing side marked \( x \).

Decide which sides you are given in terms of the acute angle given (never use the \( 90^\circ \) angle!!) In this case,
- we’re given an acute angle of \( 37^\circ \).
- The side marked \( x \) is the **leg adjacent** to the \( 37^\circ \) angle.
- The side marked “18” is the **leg opposite** the \( 37^\circ \) angle.
- This means that we will need to use the tangent ratio.

\[
\tan 37^\circ = \frac{18}{x}
\]

Cross-multiply and solve for \( x \):

\[
x \tan 37^\circ = 18 \\
x = \frac{18}{\tan 37^\circ}
\]

Your calculator “knows” all the trig ratios, so you can just type in “18/tan(37)” and you will get your answer! Round to whatever the problem dictates.

\[ x \approx 23.89 \]

**Find the measure of each side indicated. Round your answers to the nearest tenth.**

![Diagram](image)
You Try it!

Practice Problems: Determine the length of side \( x \) and \( y \) of each right triangle using trigonometric ratios.

1) 

Concept 4: Applications of Trig Problems

<table>
<thead>
<tr>
<th>Angle of depression</th>
<th>horizontal line</th>
<th>viewpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>when looking down at an object, the angle whose sides are your line of sight and a horizontal line through your viewpoint</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of elevation</th>
<th>horizontal line</th>
</tr>
</thead>
<tbody>
<tr>
<td>when looking up at an object, the angle whose sides are your line of sight and a horizontal line through your viewpoint</td>
<td></td>
</tr>
</tbody>
</table>

Example:

A lamp points toward the base of a doorway as shown below.

\[ \text{Note: The figure is not drawn to scale.} \]

What is the height, in feet, of the lamp?
Concept 3: Using Inverse Trig to find Missing Angles

An Inverse function is a function that “undoes” a given function. You are already familiar with some functions and their inverse functions:

+ “undoes” –  ÷ “undoes” ×  √ “undoes” x²

Each trigonometric function has a function that “undoes” it.

<table>
<thead>
<tr>
<th>Arcsin</th>
<th>7rcos</th>
<th>Arctan</th>
</tr>
</thead>
<tbody>
<tr>
<td>is pronounced “arc sine” and is the inverse of sine. On your calculator, it is written as ( \sin^{-1} ) and is usually called “inverse sine.”</td>
<td>is pronounced “arc cos” and is the inverse of sine. On your calculator, it is written as ( \cos^{-1} ) and is usually called “inverse cosine.”</td>
<td>is pronounced “arc tan” and is the inverse of tangent. On your calculator, it is written as ( \tan^{-1} ) and is usually called “inverse tangent.”</td>
</tr>
</tbody>
</table>

It is confusing that each operation has two names and two notations!!

The purpose of the inverse functions is that it will give you the measure of the angle if you know the value of the trig ratio.

Example: Find the missing angle marked \( x \).

Decide which sides you are given in terms of the acute angle you’re interested in.

- In this case, we’re given 8, the opposite leg to angle \( x \).
- We’re also given 18, the hypotenuse.
- This means we need the sine ratio.

\[ \sin x = \frac{8}{18} \]

Now you know the sine ratio. It’s 8/18. You can leave it as a fraction or convert it to a decimal if you like. The way to “cancel out” sin is with the inverse sine function. Take the “inverse sin” of both sides

\[ \sin^{-1}(\sin x) = \sin^{-1}\left(\frac{8}{18}\right) \]

The \( \sin^{-1} \) and the sin on the left side cancel out, leaving you with just \( x \).

\[ \sin^{-1}(\sin x) = \sin^{-1}\left(\frac{8}{18}\right) \]

\[ x = \sin^{-1}\left(\frac{8}{18}\right) \]

It’s written on the calculator as \( \sin^{-1} \) and is obtained by pressing “2nd sin”. Each function has its own reciprocal trig function.

\[ x \approx 23.39° \]
Evaluate with a calculator (make sure you are in degrees mode). Round answer to nearest degree.

\[
\begin{align*}
\text{b) } \sin^{-1} (0.62) &= \\
\text{b) } \tan^{-1} (1.2) &= \\
\text{c) } \cos^{-1} \frac{5}{13} &= \\
\end{align*}
\]

Find the measure of the indicated angle to the nearest degree.

\[m \angle = \text{_____} + \text{_____} = _____
\]

Practice: Solve for the missing angle.

3.

\[m \angle = \text{_____} + \text{_____} = _____
\]

4.

\[m \angle = \text{_____} + \text{_____} = _____
\]

5.

\[m \angle = \text{_____} + \text{_____} = _____
\]

\[m \angle = 90 - \text{_____} = _____
\]
1. The angle of depression from the top of a tower to a boulder on the ground is 38°. If the tower is 25 m high, how far from the base of the tower is the boulder?

2. A hot air balloon hovers 75 feet above the ground. The balloon is tethered to the ground with a rope that is 125 feet long. At what angle of elevation, \( E \), is the rope attached to the ground? Round your answer to the nearest degree.

Challenge Problem:
The accompanying diagram shows a flagpole that stands on level ground. Two cables, \( r \) and \( s \), are attached to the pole at a point 16 feet above the ground. The combined length of the two cables is 50 feet. If cable \( r \) is attached to the ground 12 feet from the base of the pole, what is the measure of the angle, \( x \), to the nearest degree, that cable \( s \) makes with the ground?
SUMMARY of TRIG NOTES OVERALL

Trig Functions:

\[
\begin{align*}
\text{Sine} & : \quad \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \\
\text{Cosine} & : \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \\
\text{Tangent} & : \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}
\end{align*}
\]

All Trig functions are used for right triangles only. Each one is a proportion and when working the problem should be cross multiplied and solved.

\[
\begin{align*}
\text{Sine} \quad \sin \theta & = \frac{\text{length leg opposite of angle}}{\text{length of hypotenuse}} \\
\text{Cosine} \quad \cos \theta & = \frac{\text{length leg adjacent of angle}}{\text{length of hypotenuse}} \\
\text{Tangent} \quad \tan \theta & = \frac{\text{length leg opposite of angle}}{\text{length leg adjacent of angle}}
\end{align*}
\]

If you use a different angle, then the adjacent and opposite legs reverse. You never use the right angle for trig and the hypotenuse never changes position.

Mnemonic for remember the trig functions

SOHCAHTOA

\[
\begin{align*}
\text{Sine} & : \quad \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \\
\text{Cosine} & : \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \\
\text{Tangent} & : \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}
\end{align*}
\]

Since Trig is done with angles and angles are measured in degrees, then the calculator set in the right mode. If the calculator is not set to degree mode, then every answer will be wrong.

Calculators have 3 modes; grad, rad, and deg. The deg mode is the degree mode. In TI graphing calculators the mode is found using the mode key, and the second line you will find the deg. Select it by using the equal(enter) key. If you have a different calculator, then consult the manual or ask a math teacher to help you. Each calculator can be different, so it is always good to know how your calculator works.

There are two types of trig problems. One you find the missing side, the other your find the missing angle. Each one is worked a different way, so look at the examples carefully.

Example 1  Find the missing angle

\[
\begin{align*}
\text{Because the hypotenuse and side adjacent to the angle is given, the trig function using those pieces is cosine. This allows us to set up the following equation.} \\
\text{Though it looks like a proportion, do not solve it like a proportion.}
\end{align*}
\]

\[
\cos \theta = \frac{3}{12}
\]

To solve this find $\cos^{-1}$, then type in fraction (3/12). Hit enter, since answer is an angle round to nearest degree. $\theta = 76^\circ$
Example 2  Using Sine
Find x.

Because the hypotenuse and the side opposite the $21^\circ$ angle is given, the trig function using those 2 pieces of information is sine. This allows use to set up the following equation. Once the equation is set, then solve the proportion.

$$\sin 21^\circ = \frac{x}{12}$$  Equation

$$x = 12 \cdot (\sin 21^\circ)$$  cross multiply

$$x = 12 \cdot (0.3584)$$  find sine of 21 degrees

$$x = 4.3004$$  multiply, round to the nearest ten thousandths

Example 3  Using Cosine
Find x.

Because the hypotenuse and the side adjacent the $42^\circ$ angle is given, the trig function using those 2 pieces of information is cosine. This allows use to set up the following equation. Once the equation is set, then solve the proportion.

$$\cos 42^\circ = \frac{20}{x}$$  Equation

$$x(\cos 42^\circ) = 20$$  cross multiply

$$x = \frac{20}{(\cos 42^\circ)}$$  divide each side by cosine 42 degrees

$$x = \frac{20}{(0.7431)}$$  find cosine of 42 degrees

$$x = 26.9127$$  divide, round to the nearest ten thousandths

Example 4  Using Tangent
Find x.

Because the side opposite and the side adjacent to the $63^\circ$ angle is given, the trig function using those 2 pieces of information is tangent. This allows use to set up the following equation. Once the equation is set, then solve the proportion.

$$\tan 63^\circ = \frac{x}{8}$$  Equation

$$x = 8 \cdot (\tan 63^\circ)$$  cross multiply

$$x = 8 \cdot (1.9626)$$  find tangent of 63 degrees

$$x = 15.7009$$  multiply, round to the nearest ten thousandths
Concept 1: Trig Ratios

Write the ratio that represents the trigonometric function in simplest form.

1) \( \tan A \)

2) \( \cos C \)

3) \( \sin Z \)

4) \( \sin C \)

5) \( \sin C \)

6) \( \sin C \)

7) \( \cos A \)

8) \( \cos X \)
9) \( \cos A \)

![Diagram](image1)

10) \( \cos Z \)

![Diagram](image2)

11) \( \sin Z \)

![Diagram](image3)

12) \( \sin C \)

![Diagram](image4)

13) \( \cos X \)

![Diagram](image5)

14) \( \tan A \)

![Diagram](image6)

15) \( \tan X \)

![Diagram](image7)

16) \( \tan X \)

![Diagram](image8)
Concept 2: Finding Missing Sides of Right Triangles

Directions: In problems 1 through 3, determine the trigonometric ratio needed to solve for the missing side and then use this ratio to find the missing side.

1) In right triangle ABC, \( \angle A = 58^\circ \) and \( AB = 8 \). Find the length of each of the following. Round your answers to the nearest tenth.

   (a) \( AC \)  
   (b) \( BC \)

2) In right triangle ABC, \( \angle B = 44^\circ \) and \( AB = 15 \). Find the length of each of the following. Round your answers to the nearest tenth.

   (a) \( AC \)  
   (b) \( BC \)

3) In right triangle ABC, \( \angle C = 32^\circ \) and \( AB = 24 \). Find the length of each of the following. Round your answers to the nearest tenth.

   (a) \( AC \)  
   (b) \( BC \)
Concept 3: Finding Missing Angles of Right Triangles

1) For the following right triangles, find the measure of each angle, \( x \), and \( y \), to the nearest degree:

(a) \[ \begin{array}{c}
39 \\
27 \\
x \\
\end{array} \]

(b) \[ \begin{array}{c}
11 \\
19 \\
x \\
\end{array} \]

(c) \[ \begin{array}{c}
21 \\
36 \\
x \\
y \\
\end{array} \]

(d) \[ \begin{array}{c}
51 \\
29 \\
x \\
y \\
\end{array} \]

2) Given the following right triangle, which of the following is closest to \( m\angle A \)?

(1) 28°  (3) 62°
(2) 25°  (4) 65°

3) In the diagram shown, \( m\angle N \) is closest to

(1) 51°  (3) 17°
(2) 54°  (4) 39°
Concept 4: Trig Applications

1. A flagpole that is 45 feet high casts a shadow along the ground that is 52 feet long. What is the angle of elevation, \( A \), of the sun? Round your answer to the nearest degree.

2. A 14 foot ladder is leaning against a house. The angle formed by the ladder and the ground is 72°.
   (a) Determine the distance, \( d \), from the base of the ladder to the house. Round to the nearest foot.
   
   (b) Determine the height, \( h \), the ladder reaches up the side of the house. Round to the nearest foot.

3. A 20-foot ladder leaning against a vertical wall reaches to a height of 16 feet. Find the sine, cosine, and tangent values of the angle that the ladder makes with the ground.

4. An access ramp reaches a doorway that is 2.5 feet above the ground. If the ramp is 10 feet long, what is the sine of the angle that the ramp makes with the ground?
HW ANSWERS

1) \( \tan A = \frac{30}{16} = \frac{15}{8} \)

2) \( \cos C = \frac{12}{15} = \frac{4}{5} \)

3) \( \sin Z = \frac{12}{20} = \frac{3}{5} \)

4) \( \sin C = \frac{30}{50} = \frac{3}{5} \)

5) \( \sin C = \frac{48}{50} = \frac{24}{25} \)

6) \( \sin C = \frac{14}{50} = \frac{7}{25} \)

7) \( \cos A = \frac{3}{5} \)

8) \( \cos X = \frac{24}{30} = \frac{4}{5} \)

9) \( \cos A = \frac{36}{45} = \frac{4}{5} \)

10) \( \cos Z = \frac{15}{25} = \frac{3}{5} \)

11) \( \sin Z = \frac{18}{30} = \frac{3}{5} \)

12) \( \sin C = \frac{36}{45} = \frac{4}{5} \)

13) \( \cos X = \frac{8}{17} \)

14) \( \tan A = \frac{30}{40} = \frac{3}{4} \)

15) \( \tan X = \frac{24}{10} = \frac{4}{5} \)

16) \( \tan X = \frac{15}{20} = \frac{3}{4} \)

Directions: In problems 1 through 7, determine the trigonometric ratio needed to solve for the missing side and then use this ratio to find the missing side.

1) In right triangle \( \triangle ABC \), \( \angle A = 58^\circ \) and \( AB = 8 \). Find the length of each of the following. Round your answers to the nearest tenth.

(a) \( AC \) 
\[
\frac{\cos(58^\circ)}{1} = \frac{A}{H} \\
\cos(58^\circ) = \frac{A}{H} \\
8 = AC \cdot \cos(58^\circ) \\
AC = \frac{8}{\cos(58^\circ)} \\
15.1 \approx AC
\]

(b) \( BC \) 
\[
\frac{\tan(58^\circ)}{1} = \frac{O}{A} \\
\tan(58^\circ) = \frac{O}{A} \\
BC = 8 \cdot \tan(58^\circ) \\
BC \approx 12.8
\]

2) In right triangle \( \triangle ABC \), \( \angle A = 44^\circ \) and \( AB = 15 \). Find the length of each of the following. Round your answers to the nearest tenth.

(a) \( AC \) 
\[
\frac{\sin(44^\circ)}{1} = \frac{O}{H} \\
\sin(44^\circ) = \frac{O}{H} \\
AC = 15 \cdot \sin(44^\circ) \\
AC \approx 10.4
\]

(b) \( BC \) 
\[
\frac{\cos(44^\circ)}{1} = \frac{A}{H} \\
\cos(44^\circ) = \frac{A}{H} \\
BC = 15 \cdot \cos(44^\circ) \\
BC \approx 10.8
\]
3) In right triangle $ABC$, $m \angle C = 32^\circ$ and $AB = 24$. Find the length of each of the following. Round your answers to the nearest tenth.

(a) $AC$

\[
\tan(32) = \frac{24}{AC} = \frac{0.6249}{AC} \\
AC = \frac{24}{\tan(32)} \\
AC \approx 38.4
\]

(b) $BC$

\[
\tan(32) = \frac{24}{BC} = \frac{0.6249}{BC} \\
BC = \frac{24}{\tan(32)} \\
BC \approx 45.3
\]

2) Given the following right triangle, which of the following is closest to $m \angle A$?

(i) $22^\circ$

(ii) $25^\circ$

(iii) $35^\circ$

(iv) $55^\circ$

3) In the diagram shown, $m \angle N$ is closest to

(i) $5^\circ$

(ii) $17^\circ$

(iii) $34^\circ$

(iv) $39^\circ$

A flagpole that is 45 feet high casts a shadow along the ground that is 52 feet long. What is the angle of elevation, $A$, of the sun? Round your answer to the nearest degree.

\[
m \angle A = \tan^{-1}\left(\frac{45}{52}\right) \approx 41^\circ
\]

Part A

\[
\begin{align*}
\cos(72) & = \frac{a}{h} \\
\cos(72) & = \frac{14}{d} \\
d - 14 & = \cos(72) \\
d & = 4 \text{ ft}
\end{align*}
\]

Part B

\[
\begin{align*}
\sin(72) & = \frac{b}{h} \\
\sin(72) & = \frac{1}{14} \\
h & = 14 \cdot \sin(72) \\
h & \approx 13 \text{ ft}
\end{align*}
\]

5) A 20-foot ladder leaning against a vertical wall reaches a height of 16 feet. Find the sine, cosine, and tangent values of the angle that the ladder makes with the ground.

\[
\begin{align*}
20^2 & = a^2 + 16^2 \\
144 & = a^2 \\
\sqrt{144} & = a \\
a & = 12 \\
\sin x & = \frac{O}{H} = \frac{16}{20} = \frac{4}{5} \\
\cos x & = \frac{A}{H} = \frac{12}{20} = \frac{3}{5} \\
\tan x & = \frac{O}{A} = \frac{16}{12} = \frac{4}{3}
\end{align*}
\]

4) An access ramp reaches a doorway that is 2.5 feet above the ground. If the ramp is 10 feet long, what is the sine of the angle that the ramp makes with the ground?

\[
\begin{align*}
\sin x & = \frac{O}{H} = \frac{2.5}{10} = \frac{1}{4}
\end{align*}
\]
Day 2: Chapter 9-2: Angles and Arcs as Rotations

Warm Up

Determine the trigonometric ratios for the following triangle:

(a) \( \sin A = \) 
(b) \( \cos A = \) 
(c) \( \tan A = \) 
(d) \( \sin B = \) 
(e) \( \cos B = \) 
(f) \( \tan B = \)

Angles, pretty much from now on, are going to be considered as drawn on the coordinate plane.

Terminology: Angles in the 4 Quadrants

- One side of the angle is drawn on the positive side of the \( x \)-axis, beginning at the origin. This side is called the initial side.
- The other side of the angle is called the terminal side (terminal means “ending”) of the angle.

- An angle formed by a counter clockwise rotation has a ______________.

- An angle formed by a clockwise rotation has a _________________.

[Diagram of angles in different quadrants and clockwise and counter clockwise rotations]
- If an angle terminates in a quadrant, it is named after the quadrant it lands in (ie “QI angle,” “QII angle,” “QIII angle”)
- Angles are usually going to be represented by the Greek letter theta:
- If the terminal side terminates on the boundary of a quadrant, it is called a quadrantal angle. Any integer multiple of 90° is a quadrantal angle: ______, ______, ______, ______.

**Classifying Angles by Quadrant**

![Diagram of quadrants with angle measures]

- ___ < θ < ___
- ___ < θ < ___
- ___ < θ < ___
- ___ < θ < ___

**Concept 1: Drawing Angles in Standard Position.**

Draw an angle with the given measure in standard position and determine the quadrant in which the angle lies.

1. 145°

![145° angle in standard position]

2. 270°

![270° angle in standard position]

3. 500°

![500° angle in standard position]

4. -50°

![-50° angle in standard position]
You Try It!

Draw an angle with the given measure in standard position and determine the quadrant in which the angle lies.

1. $60^\circ$

2. $210^\circ$

3. $450^\circ$

4. $-40^\circ$
Concept 2: Finding Coterminal Angles

- Angles which terminate in the same exact place are called **coterminal** angles. To find an angle coterminal with a given angle, simply add or subtract multiples of _____.

Another way of explaining is that Coterminal angles are angles in standard position (angles with the initial side on the positive x-axis) that have a common terminal side. For example 30°, –330° and 390° are all coterminal. (look below). Here –330° is the negative coterminal angle of 30° and 390° is positive coterminal angle of 30°.

We can use the formula Coterminal angle = A + 360n; where A is the angle and n is the number of complete 360° rotations of the terminal ray.

**If two angles are coterminal then**
Example 1: Find one angle with positive measure and one angle with negative measure coterminal with each given angle.

Practice 1: Find the angle of smallest positive measure that is coterminal with an angle of the given measure.

1) 910°  
2) 200°  
3) -140°
Regents questions

1. In which quadrant does a -285° angle lie?

   (1) I
   (2) II
   (3) III
   (4) IV

Explain your answer below.

2. Which angle is not coterminal with an angle that measures 300°?

   (1) -420°
   (2) -300°
   (3) -60°
   (4) 660°

Explain your answer below.

3. Which angle is coterminal with an angle that measures -120°?

   (1) -80°
   (2) 60°
   (3) 240°
   (4) 580°

Explain your answer below.
Challenge

ENTERTAINMENT  Suppose the gondolas on the Navy Pier Ferris wheel were numbered from 1 through 40 consecutively in a counterclockwise fashion. If you were sitting in gondola number 3 and the wheel were to rotate counterclockwise through 846 degrees, which gondola used to be in the position that you are in now?

SUMMARY

Draw an angle with the given measure in standard position.

a. $240^\circ$  
$240^\circ = 180^\circ + 60^\circ$

Draw the terminal side of the angle $60^\circ$ counterclockwise past the negative x-axis.

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

a. $240^\circ$

A positive angle is $240^\circ + 360^\circ$ or $600^\circ$.

A negative angle is $240^\circ - 360^\circ$ or $-120^\circ$.

Exit Ticket

Which angle is coterminal with an angle of $125^\circ$?

[A] $425^\circ$  [B] $-125^\circ$

[C] $-235^\circ$  [D] $235^\circ$
Angles and Arcs as Rotations

State if the given angles are coterminal.

1) 50°, -310°
2) 30°, 750°
3) 15°, -375°
4) 65°, 605°
5) 0°, 360°
6) 80°, 440°

Find a coterminal angle between 0° and 360°.

7) 692°
8) 531°
9) 712°
10) 710°
11) 575°
12) 435°

Find a positive and a negative coterminal angle for each given angle.

13) -60°
14) -290°
15) 176°
16) 0°
17) 250°
18) -315°

State the quadrant in which the terminal side of each angle lies.

19) 20°
20) 75°
21) -323°
22) -255°
23) 330°
24) -125°
25) 130°
26) 200°
27) 6°
28) 340°
29) -60°
30) 140°
Find the measure of each angle.

31) 

32) 

33) 

34) 

35) 

36) 

37) 

38)
Draw an angle with the given measure in standard position.

39) \(-160^\circ\)

40) \(250^\circ\)

41) \(320^\circ\)

42) \(-240^\circ\)

43) \(10^\circ\)

44) \(-320^\circ\)
Answers to Angles and Arcs as Rotations (ID: 1)

1) Yes  
5) Yes  
9) 352°  
13) 300° and −420°  
17) 610° and −110°  
21) I  
25) II  
29) IV  
33) −300°  
37) 350°  

2) Yes  
6) Yes  
10) 350°  
14) 70° and −650°  
18) 45° and −675°  
22) II  
26) III  
30) IV  
34) 30°  
38) −290°  

3) No  
7) 332°  
11) 215°  
15) 536° and −184°  
19) I  
23) IV  
27) I  
31) −60°  
35) −210°  
39)  

4) No  
8) 171°  
12) 75°  
16) 360° and −360°  
20) I  
24) III  
28) IV  
32) 175°  
36) −140°  
40)
Day 3: ch 9-3 The Unit Circle, Sine, and Cosine and Ch. 9-4 The Tangent function

Warm – Up:

Which angle is coterminal with an angle that measures \(-50^\circ\)?

(1) \(-300^\circ\)
(2) \(290^\circ\)
(3) \(160^\circ\)
(4) \(670^\circ\)

Explain your answer below.

The Unit Circle: A circle centered at the origin with a radius of 1.
In terms of angles as rotations,

If \(P(x, y)\) are the coordinates of any point on the unit circle, and \(\theta\) is the angle of rotation from the x-axis to point \(P\), then

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}
\]

Thus, every point on the unit circle can also be written as

Concept 1: Finding the Trig Values of Points on Unit Circle

Examples:
In questions 1 -3 you are given the coordinates of point P, m<ROP =θ and OR =1. Find a) sin θ  b) cos θ  c)tanθ

1. P$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

2. P$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

3. P(.6, -.8)
You Try it!

Given points on a unit circle.
Find a) \( \sin \theta \)  
  b) \( \cos \theta \)  
  c) \( \tan \theta \)

11. \( P\left(\frac{3}{5}, \frac{4}{5}\right) \) 
12. \( P\left(\frac{12}{13}, -\frac{5}{13}\right) \) 
13. \( P\left(\frac{8}{17}, \frac{15}{17}\right) \)

As point \( P(x, y) \) moves around the unit circle, and \( \theta \) increases from 0° to 360°, \( x \) and \( y \) change signs, and thus the signs of \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \) also change.

**Signs of Trig Functions in the Quadrants**

<table>
<thead>
<tr>
<th>Quadrant II</th>
<th>Quadrant III</th>
<th>Quadrant IV</th>
</tr>
</thead>
</table>
| x is _____ and y is _____  
\( \therefore \) \( \cos \theta \) is ___ and \( \sin \theta \) is ___.  |
| x is _____ and y is _____  
\( \therefore \) \( \cos \theta \) is ___ and \( \sin \theta \) is ___.  |
| x is _____ and y is _____  
\( \therefore \) \( \cos \theta \) is ___ and \( \sin \theta \) is ___.  |
Signs of Trigonometric Functions

<table>
<thead>
<tr>
<th></th>
<th>QI</th>
<th>QII</th>
<th>QIII</th>
<th>QIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tangent</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There is an easy way to remember the signs of sin, cos, and tan in the different quadrants.

_____ is/are + in QI
_____ is/are are + in QII
_____ is/are are + in QIII
_____ is/are are + in QIV

Concept 2: Determine the sign (+/-) of trig functions on the coordinate plane.

IMPORTANT: “>0” means “is positive” “<0” means “is negative”

Example 2: In what quadrant(s) could $\theta$ be when…

a) $\sin \theta > 0$ and $\cos \theta > 0$  
   
   
   b) $\sin \theta > 0$ and $\cos \theta < 0$

   
   
   c) $\sin \theta > 0$ and $\tan \theta > 0$

   
   
   d) $\sin \theta > 0$ and $\tan \theta < 0$

   
   
   e) $\tan \theta > 0$ and $\cos \theta > 0$

   
   
   f) $\tan \theta > 0$ and $\cos \theta < 0$
Concept 3: Finding Sine, Cosine, and Tangent values given a point in a quadrant.

- Determine what quadrant the point lies in.
- Draw an approximate location of the point, connecting it to the origin.
- **Draw a right triangle connecting the point to the x-axis. NEVER THE Y-AXIS!!**
- Label the sides of the triangle appropriately, using the values of the point. If necessary, use the Pythagorean Theorem to find the 3\text{rd} side (in simplest radical form, if possible.)
- The angle $\theta$ ALWAYS is written between the hypotenuse and the x-axis.

**Key Concept**

**Trigonometric Functions, $\theta$ in Standard Position**

Let $\theta$ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of $\theta$. Using the Pythagorean Theorem, the distance $r$ from the origin to $P$ is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.

\[
\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}, \quad \tan \theta = \frac{y}{x}, \quad x \neq 0
\]

Draw each of the following points on a coordinate plane. Let $\theta$ be the angle in standard position that terminates at that point. Determine the sine, cosine, and tangent of $\theta$.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (5, 12)</td>
<td>2. (-8, 15)</td>
</tr>
</tbody>
</table>
Let's put this all together!

Example 4: Let point P be on the terminal side of $\theta$. Draw a picture, and determine the sine, cosine, and tangent of the angle.

a. If $\sin \theta = \frac{12}{13}$, where $\theta$ is in Quadrant I, find $\cos \theta$ and $\tan \theta$.

b. If $\cos \theta = \frac{2}{3}$, where $\theta$ is in Quadrant IV, find $\sin \theta$ and $\tan \theta$.

c. If $\tan \theta = 3$, where $\theta$ is in Quadrant III, find $\sin \theta$ and $\cos \theta$.

d. If $\sin \theta = \frac{5}{6}$, where $\theta$ is in Quadrant II, find $\tan \theta$ and $\cos \theta$. 
**SUMMARY**

**Example 1 Evaluate Trigonometric Functions for a Given Point**

Find the exact values of the six trigonometric functions of $\theta$ if the terminal side of $\theta$ contains the point $(5, -12)$.

From the coordinates given, you know that $x = 5$ and $y = -12$. Use the Pythagorean Theorem to find $r$.

$$r = \sqrt{x^2 + y^2}$$  \hspace{1cm} \text{Pythagorean Theorem}

$$= \sqrt{5^2 + (-12)^2}$$  \hspace{1cm} \text{Replace } x \text{ with } 5 \text{ and } y \text{ with } -12.

$$= \sqrt{169} \text{ or } 13$$  \hspace{1cm} \text{Simplify.}

Now, use $x = 5$, $y = -12$, and $r = 13$ to write the ratios.

$$\sin \theta = \frac{y}{r}$$

$$= \frac{-12}{13} \text{ or } \frac{-12}{13}$$

$$\cos \theta = \frac{x}{r}$$

$$= \frac{5}{13}$$

$$\tan \theta = \frac{y}{x}$$

$$= \frac{-12}{5} \text{ or } \frac{-12}{5}$$

**Example 2 Find Sine and Cosine Given Point on Unit Circle**

Given an angle $\theta$ in standard position, if $P\left(\frac{2\sqrt{2}}{3}, \frac{-1}{3}\right)$ lies on the terminal side and on the unit circle, find $\sin \theta$ and $\cos \theta$.

$$P\left(\frac{2\sqrt{2}}{3}, \frac{-1}{3}\right) = P(\cos \theta, \sin \theta),$$

so $\sin \theta = \frac{-1}{3}$ and $\cos \theta = \frac{2\sqrt{2}}{3}$.

**Exit Ticket**

If $\sin \theta$ is negative and $\cos \theta$ is negative, in which quadrant does the terminal side of $\theta$ lie?

1) I
2) II
3) III
4) IV
Day 3 - HW

1. Fill in the table with the sign of the function in each quadrant.

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>sin θ</th>
<th>cos θ</th>
<th>tan θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. If the coordinates of point P are \((0.5, 0.5\sqrt{3})\), find
   a. sin θ
   b. cos θ
   c. tan θ

3. If the coordinates of point P are \(\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)\), find
   a. sin θ
   b. cos θ
   c. tan θ

4. Given m∠A = 250°.
   a. In what quadrant does the terminal side of ∠A lie?
   b. Is sin A positive or negative? Why?

   In 5–11, select the numeral preceding the word or expression that best answers the question.

5. If sin θ > 0 and cos θ < 0, in which quadrant does θ lie?
   (1) I
   (2) II
   (3) III
   (4) IV

6. If tan θ is positive and cos θ is negative, in what quadrant does θ terminate?
   (1) I
   (2) II
   (3) III
   (4) IV

7. If sin B = \(-\frac{3}{5}\) and cos B > 0, in what quadrant does ∠B lie?
   (1) I
   (2) II
   (3) III
   (4) IV

8. If tan A > 0 and (tan A)(sin A) > 0, in what quadrant does ∠A lie?
   (1) I
   (2) II
   (3) III
   (4) IV

9. If tan x = -1 and cos x = -\frac{\sqrt{2}}{2}, in what quadrant could angle x terminate?
   (1) I
   (2) II
   (3) III
   (4) IV

10. If sin θ = \(-\frac{1}{2}\) and cos θ = -\frac{\sqrt{3}}{2}, which of the following could be the measure of θ?
    (1) 30°
    (2) 150°
    (3) 210°
    (4) 330°

11. Which of the following could be true?
    (1) sin 300° = \frac{\sqrt{3}}{2}
    (2) sin 240° = \frac{\sqrt{3}}{2}
    (3) sin 120° = \frac{\sqrt{3}}{2}
    (4) sin 60° = -\frac{\sqrt{3}}{2}
In 12–15, use the figure below, where $OA = 1$, and $m \angle AOB = \theta$.

12. If the coordinates of point $A$ are $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$, find
   a. $\sin \theta$
   b. $\cos \theta$
   c. $\tan \theta$

13. Name a line segment whose directed distance is the value of
   a. $\sin \theta$
   b. $\cos \theta$
   c. $\tan \theta$

14. If the coordinates of point $A$ are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\theta = 30^\circ$, find $\cos 30^\circ$.

15. If the coordinates of point $A$ are $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $\theta = 45^\circ$, find $\sin 45^\circ$.

---

In 13–20, $P$ is a point on the terminal side of an angle in standard position with measure $\theta$ and on a circle with center at the origin and radius $r$. For each point $P$, find: a. $r$ b. $\cos \theta$ c. $\sin \theta$ d. $\tan \theta$

13. $(7, 24)$
14. $(8, 15)$
15. $(-1, -1)$
16. $(-3, -4)$
Answers to Unit Circle HW

1.

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>Sin θ</th>
<th>Cos θ</th>
<th>Tan θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>II</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>III</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>IV</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
</tbody>
</table>

2. a. \(0.5\sqrt{3}\)  
b. 0.5  
c. \(\sqrt{3}\)

3. a. \(\frac{\sqrt{2}}{2}\)  
b. \(\frac{\sqrt{2}}{2}\)  
c. 1

4. a. III  
b. Negative, because \(y\) is negative.

5. 2  
6. 3  
7. 4  
8. 1  
9. 2  
10. 3  
11. 3

12. a. \(\frac{\sqrt{5}}{3}\)  
b. \(\frac{2}{3}\)  
c. \(\frac{\sqrt{5}}{2}\)

13. a. \(\overline{AB}\)  
b. \(\overline{OB}\)  
c. \(\overline{CD}\)

14. \(\frac{1}{2}\)  
15. \(\frac{\sqrt{2}}{2}\)

13. a. 25  
b. \(\frac{7}{25}\)  
c. \(\frac{24}{25}\)  
d. \(\frac{24}{7}\)

14. a. 17  
b. \(\frac{8}{17}\)  
c. \(\frac{15}{17}\)  
d. \(\frac{15}{8}\)

15. a. \(\sqrt{2}\)  
b. \(-\frac{\sqrt{2}}{2}\)  
c. \(-\frac{\sqrt{2}}{2}\)  
d. 1

16. a. 5  
b. \(-\frac{3}{5}\)  
c. \(-\frac{4}{5}\)  
d. \(\frac{4}{5}\)
Day 5: Chapter 9-6: “Special” Angles, Exact Trig Values/Function Values from the Calculator

Warm – Up

The diagram below shows right triangle UPC.

Which ratio represents the sine of ∠U?

1) \( \frac{15}{8} \) 2) \( \frac{15}{17} \)
3) \( \frac{8}{15} \) 4) \( \frac{8}{17} \)

Which ratio represents \( \cos A \) in the accompanying diagram of \( \triangle ABC \)?

1) \( \frac{5}{13} \) 2) \( \frac{12}{13} \) 3) \( \frac{12}{5} \) 4) \( \frac{13}{5} \)

“Special” Right Triangles

30 – 60 – 90° Triangle
(AKA ½ of an equilateral triangle)

Assume the length of the side of the equilateral \( \Delta = 2 \).

Use these triangles to determine the following trigonometric values:

<table>
<thead>
<tr>
<th></th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosine</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tangent</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Other “Special” values
We already know that on the unit circle, \( x = \cos \theta \) and \( y = \sin \theta \), and that \( \tan \theta = \frac{\sin \theta}{\cos \theta} \), so we can use that knowledge to determine the trig values of quadrantal angles.

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>90°</th>
<th>180°</th>
<th>270°</th>
<th>360°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosine</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Tangent</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Putting it all together (only QI)

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosine</td>
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<tr>
<td>Tangent</td>
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</tr>
</tbody>
</table>

How to construct this table:
- For Sines and Cosines only, write a denominator of “2” for each.
- For Sine, fill in the following numerators, left to right: \( \sqrt{0}, \sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4} \).
- For Cosine, fill in the following numerators, left to right: \( \sqrt{4}, \sqrt{3}, \sqrt{2}, \sqrt{1}, \sqrt{0} \).
- Simplify.
- Since tangent = sin/cos, each tangent box is sin/cos. Divide, and rationalize the denominators.

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Cosine</td>
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<td></td>
</tr>
<tr>
<td>Tangent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Find the EXACT value of each expression.

a) \((\sin 30^\circ)(\cos 60^\circ)\)  
b) \(\cos 60^\circ + 3 \tan 45^\circ\)  
c) \((\sin 60^\circ)(\tan 30^\circ)\)  
d) \((\sin 45^\circ)^2 + (\cos 45^\circ)^2\)  
e) \(\sin 30^\circ + \cos 30^\circ\)  
f) \(2 \cos 30^\circ + 4 \tan 60^\circ\)  
g) Let \(f(x) = \sin 2x\).  
Determine \(f(30^\circ)\)  
h) \(\cos 30^\circ/\sin 45^\circ\)  
i) \(\frac{\sin 30^\circ}{\cos 30^\circ}\)
Algebra2/Trig: Finding Angle Measures to the Nearest Degree

When asked to find angle measures to the nearest degree, always give the answer rounded to the nearest ten-thousandth (4 decimal places).

- Angles can also be measured more precisely than the nearest degree.
- Degrees are divided into 60 minutes, symbolized by ‘.
- Each minute is divided into 60 seconds, symbolized with a “.

So a degree measure could be given as: 23°31’14” (read “23 degrees 31 minutes 14 seconds”).

You can use your calculator to perform functions on these measurements in 1 of 2 ways:

**Example: What is \( \sin 23°31’14” \)?**

1. Use the degree, minutes, seconds (DMS or D°M’S”) part of your calculator:

   Type: \( \sin 23^\circ 31’14” \)

2. You know there are 60 minutes in 1 degree and 60 seconds in 1 minute, so write it as an expression:

   Type: \( \sin 23 + 31 / 60 + 14 / 3600 \) (where did 3600 come from?)

To work backwards, given an angle measure in DMS form, enter the given angle measure and press: \(^{2\text{nd}}\), ANGLE (APPS btn), 4, ENTER. This should put ►DMS after your decimal degree angle measure and convert it to DMS.

**Example 2) Convert from degrees, minutes, and seconds to decimal degrees or vice versa. (3 decimal places)**

<table>
<thead>
<tr>
<th>#</th>
<th>Degrees, Minutes, Seconds</th>
<th>Decimal Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>23°30’</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>141°25’45”</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td></td>
<td>12.25°</td>
</tr>
<tr>
<td>d.</td>
<td></td>
<td>39.426°</td>
</tr>
<tr>
<td>e.</td>
<td>59°59’59”</td>
<td></td>
</tr>
<tr>
<td>f.</td>
<td></td>
<td>127.001°</td>
</tr>
</tbody>
</table>
1. A ladder 15 feet long rests against the id of a building, with the foot of the ladder 4 feet from the base of the building. Find the measure of the angle that the ladder makes with the horizontal ground: a. to the nearest degree b. to the nearest minute.

2. A standard rectangular sheet of paper measures 8 $\frac{1}{2}$ inches by 11 inches. A diagonal is drawn, connecting opposite corners of the paper. Find, to the nearest minute, the measure of the two acute angles formed by the diagonal.

3. A 20-foot ladder leans against a wall. The top of the ladder reaches 18.5 feet up the side of the building. Find the measure of the angle the ladder makes with the ground: a. to the nearest degree b. to the nearest minute c. to the nearest ten minutes.
Summary

30°-60°-90° Triangle Pattern Formulas
(you do not need to memorize these formulas as such, but you do need to memorize the relationships)

Labeling:
- \( H \) = hypotenuse
- \( LL \) = long leg (across from 60°)
- \( SL \) = short leg (across from 30°)

45°-45°-90° (Isosceles Right Triangle) Pattern Formulas
(you do not need to memorize these formulas as such, but you do need to memorize the patterns)

Exit Ticket:

The value of \( 2(\sin 30°)(\cos 30°) \) is equal to the value of:
1. \( \sin 60° \)  
2. \( \cos 60° \)  
3. \( \sin 90° \)  
4. \( \tan 30° \)
Answers to Day 5 HW

9-5 The Reciprocal Trigonometric Functions (pages 377–378)

8. a. $\frac{2}{3}$
   b. $\frac{\sqrt{5}}{3}$
   c. $\frac{2\sqrt{5}}{5}$
   d. $\frac{3\sqrt{5}}{5}$
   e. $\frac{3}{2}$
   f. $\frac{\sqrt{5}}{2}$

9. a. $\frac{3\sqrt{2}}{5}$
   b. $-\frac{\sqrt{7}}{5}$
   c. $\frac{3\sqrt{14}}{7}$
   d. $-\frac{5\sqrt{7}}{6}$
   e. $\frac{5\sqrt{2}}{6}$
   f. $\frac{\sqrt{14}}{6}$

10. a. $\frac{3}{7}$
    b. $-\frac{2\sqrt{10}}{7}$
    c. $\frac{3\sqrt{10}}{20}$
    d. $\frac{7\sqrt{10}}{20}$
    e. $\frac{7}{3}$
    f. $\frac{2\sqrt{10}}{3}$

9-6 Function Values of Special Angles (pages 380–381)

3. $\frac{\sqrt{3}}{2}$
4. $\frac{1}{2}$
5. 2
6. $\frac{\sqrt{3}}{3}$
7. $\sqrt{3}$
8. $\frac{1}{2}$
9. 2
10. $\frac{\sqrt{3}}{2}$
11. $\frac{2\sqrt{3}}{3}$
12. $\sqrt{3}$
13. $\frac{\sqrt{3}}{3}$
14. $\frac{\sqrt{2}}{2}$

15. $\sqrt{2}$
16. $\frac{\sqrt{3}}{2}$
17. $\sqrt{2}$
18. 1
19. 1
20. −1
21. −1
22. 0
23. Undefined
24. 0
25. Undefined
26. 0
27. Undefined
28. −1
29. −1
30. Undefined
31. 0
32. 1
33. 1
34. $\frac{\sqrt{3} + 1}{2}$
35. 3
36. 1
37. 1
38. $\frac{1}{4}$
39. 1
40. $\frac{1}{2}$
41. 1
42. $\frac{2\sqrt{3}}{3}$
43. 1
44. $\frac{1}{4}$

9-7 Function Values from the Calculator (pages 384–385)

Developing Skills
3. 0.4695
4. 0.8192
5. 4.7046
6. −0.1736
7. 0.1736
8. 0.3640
9. 0.3640
10. −0.2588
11. 0.2588
12. −0.9848
13. 0.9848
14. 0.2679
15. 0.2679
16. −0.5736
17. −0.5736
18. −0.1736
19. −0.1736
20. −0.1736
21. 0.9500
22. 0.8450
23. 38.1885
24. 0.9621
25. −0.1352
26. −0.9048
27. −0.4258
28. 16.1190
29. 3.2361
30. 3.8637
31. 0.5095
32. −5.7588
33. 1.2208
34. −3.7321
35. −1.1034
36. 0.2867
37. 1.6616
38. −4.4454
39. 20°
40. 64°
41. 12°
42. 40°
43. 35°
44. 87°
45. 3°
46. 62°
47. 33°
Day 6: Ch. 9-8: Reference Angles, Trig Values in All Quadrants

Warm-Up

In the diagram, the center of circle $O$ is at the origin, radius $OB = 1$, and $m \angle AOB = 30^\circ$.

What are the coordinates of point $B$?

1. $\left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right)$
2. $\left( \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$
3. $\left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right)$
4. (1, 1)

Reference Angles

We already know that we can have trigonometric values of any angle, in any quadrant, and we've already determined what the signs (+/-) of each of them are. But how can we find the actual trig function values?

A reference angle is an acute angle that is related to the given angle $\theta$. I will usually refer to the reference angle as $\theta_R$.

The sin, cos, or tan value of any angle $\theta$ is the same as its reference angle, differing only by a possible sign change. In other words, for example, $|\sin \theta| = \sin \theta_R$. 


Reference angles look different in each quadrant. In QI, the reference angle for $\theta$ is $\theta$ itself. Every angle in QI is acute, so any angle in QI ($\theta_J$) doesn’t need a reference angle.

Reference angles for other quadrants

**QII**

In QII, $\theta_R = \boxed{90^\circ}$.  

**QIII**

In QIII, $\theta_R = \boxed{270^\circ}$.  

**QIV**

In QIV, $\theta_R = \boxed{270^\circ}$.

**REMEMBER:** Reference angles are ALWAYS formed between the terminal side of the original angle and the $x$-axis. NEVER with the $y$-axis!!

Also, there are no reference angles for quadrantal angles ($0^\circ$, $90^\circ$, $180^\circ$, $270^\circ$...)

Draw each of the following angles. Determine the reference angle, and use the picture to express each of the following as a function of a positive acute angle. (This means use reference angles instead, and also include the sign of the answer. Your answers may or may not involve “special” function values.

a) $\cos 260^\circ$  
b) $\sin 715^\circ$  
c) $\tan(-110)^\circ$
d) \( \tan 170^\circ \) 

e) \( \tan 300^\circ \) 

f) \( \sin 140^\circ \) 

g) \( \cos (-100^\circ) \) 

h) \( \sin(-45^\circ) \) 

i) \( \sin 210^\circ \) 

j) \( \cos 240^\circ \) 

k) \( \tan 260^\circ \) 

l) \( \tan (-200^\circ) \) 

m) \( \cos 168^\circ 20' \) 

n) \( \sin 305^\circ 49' \) 

o) \( \tan 75^\circ 57' 17'' \)
For the following examples find the exact function value.

1) \( \cos 300^\circ \)  
2) \( \sin 240^\circ \)  
3) \( \cos 405^\circ \)

4) \( \sin 135^\circ \)  
5) \( \tan 240^\circ \)  
6) \( \cos 600^\circ \)

Find the exact value of the given expression.
7) \( \cos 135^\circ + \cos 225^\circ \)  
8) \( \sin 300^\circ + \sin (-240^\circ) \)
SUMMARY

If $\theta$ is the measure of an angle greater than $90^\circ$ but less than $360^\circ$:

<table>
<thead>
<tr>
<th>Quadrant II</th>
<th>Quadrant III</th>
<th>Quadrant IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$90^\circ &lt; \theta &lt; 180^\circ$</td>
<td>$180^\circ &lt; \theta &lt; 270^\circ$</td>
<td>$270^\circ &lt; \theta &lt; 360^\circ$</td>
</tr>
<tr>
<td>$\sin \theta = \sin (180^\circ - \theta)$</td>
<td>$\sin \theta = -\sin (\theta - 180^\circ)$</td>
<td>$\sin \theta = -\sin (360^\circ - \theta)$</td>
</tr>
<tr>
<td>$\cos \theta = -\cos (180^\circ - \theta)$</td>
<td>$\cos \theta = -\cos (\theta - 180^\circ)$</td>
<td>$\cos \theta = \cos (360^\circ - \theta)$</td>
</tr>
<tr>
<td>$\tan \theta = -\tan (180^\circ - \theta)$</td>
<td>$\tan \theta = \tan (\theta - 180^\circ)$</td>
<td>$\tan \theta = -\tan (360^\circ - \theta)$</td>
</tr>
</tbody>
</table>

Exit Ticket:

Expressed as a function of a positive acute angle, $\cos (-305^\circ)$ is equal to

(1) $-\cos 55^\circ$  
(2) $\cos 55^\circ$

(3) $-\sin 55^\circ$  
(4) $\sin 55^\circ$
Algebra2&Trig: Reference Angles

Draw an angle with the given measure in standard position. Then label, draw, and determine its reference angle.

1) 320°

2) 55°

3) 100°

4) 260°

5) 80°

6) 350°
Find the reference angle.

7) $150^\circ$

8) $205^\circ$

9)

10)

11)

12)

13)

14)
Find the exact value of each trigonometric function.

15) $\cos \theta$

16) $\sin \theta$

17) $\tan \theta$

18) $\tan \theta$

19) $\cos \theta$

20) $\tan \theta$

21) $\sin \theta$

22) $\sin \theta$
23) \( \tan \theta \)

24) \( \tan \theta \)

25) \( \cos 135^\circ \)

26) \( \cos 225^\circ \)

27) \( \cos 120^\circ \)

28) \( \tan 225^\circ \)

29) \( \sin 60^\circ \)

30) \( \tan 240^\circ \)

31) \( \sin 240^\circ \)

32) \( \tan 120^\circ \)

33) \( \sin 120^\circ \)

34) \( \cos 210^\circ \)
Answers to Algebra 2 & Trig: Reference Angles (ID: 1)

1) 90°
2) 45°
3) 30°
4) 135°
5) 225°
6) 315°
7) 270°
8) 180°

9) 85°
10) 60°
11) 45°
12) 70°
13) 65°
14) 80°
15) \frac{1}{2}
16) \frac{\sqrt{2}}{2}
17) 1
18) -\sqrt{3}
19) -\frac{\sqrt{3}}{2}
20) \frac{\sqrt{3}}{3}
21) -\frac{1}{2}
22) \frac{1}{2}
23) \sqrt{3}
24) -\frac{\sqrt{3}}{3}
25) -\frac{\sqrt{2}}{2}
26) -\frac{\sqrt{2}}{2}
27) -\frac{1}{2}
28) 1
29) \frac{\sqrt{3}}{2}
30) \sqrt{3}
31) -\frac{\sqrt{3}}{2}
32) -\sqrt{3}
Day 7: Reciprocal Trig Functions (Chapter 9-6)

Warm - Up

If an angle, in standard position, whose degree measure is 240° were sketched on the unit circle shown below, what is the exact value of sin 240°?

The Reciprocal Trig functions are literally the reciprocals of the three basic trig ratios

<table>
<thead>
<tr>
<th>The reciprocal of sine is <strong>cosecant (csc)</strong>.</th>
<th>The reciprocal of cosine is <strong>secant (sec)</strong>.</th>
<th>The reciprocal of tangent is <strong>cotangent (cot)</strong>.</th>
</tr>
</thead>
<tbody>
<tr>
<td>If ( \sin A = \frac{\text{opp}}{\text{hyp}} ) then ( \csc A = ) ( \frac{\text{hyp}}{\text{opp}} ). Also can be written as ( \csc A = )</td>
<td>If ( \cos A = \frac{\text{adj}}{\text{hyp}} ) then ( \sec A = ) ( \frac{\text{hyp}}{\text{adj}} ). Also can be written as ( \sec A = )</td>
<td>If ( \tan A = \frac{\text{opp}}{\text{adj}} ) then ( \cot A = ) ( \frac{\text{adj}}{\text{opp}} ). Also can be written as ( \cot A = )</td>
</tr>
</tbody>
</table>

Concept 1: For each find \( \csc A \), \( \sec A \), and \( \cot A \).

Given: \( \cos A = \frac{3}{5} \). \( A \) is in quadrant III

\[
\begin{align*}
\csc A & \\
\sec A & \\
\cot A & \\
\end{align*}
\]
Concept 2: Draw each of the following points on a coordinate plane. Let $\theta$ be the angle in standard position that terminates at that point. Determine all “6” trigonometric functions

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $(6, 8)$</td>
<td>2. $(-3, 3)$</td>
</tr>
</tbody>
</table>

Concept 3: Simplifying trigonometric functions in terms of $\sin \theta$, $\cos \theta$ or both

Examples:

1. Write each expression in terms of $\sin \theta$, $\cos \theta$ or both. Simplify whenever possible.
   
   a. $\tan \theta$  
   b. $\cot \theta$  
   c. $\sec \theta$  

   d. $\csc \theta$  
   e. $\sec \theta \cdot \cot \theta$  
   f. $(\tan \theta)(\csc \theta) = \sec \theta$  

   g. $\frac{\tan \theta}{\sec \theta}$  
   h. $\frac{\cot \theta}{\csc \theta}$  
   i. $\frac{\cos \theta}{\sec \theta}$  

   j. $\frac{\sin \theta}{\csc \theta}$
Concept 4: Determine the sign (+/-) of all “6” trig functions on the coordinate plane.

In 2-3, name the quadrants in which \( A \) may lie.

2. \( \cot A < 0 \)  
3. \( \sec A > 0 \)

In 4-5, name the quadrant in which \( B \) must lie.

4. \( \sec B < 0 \) and \( \tan B > 0 \)  
5. \( \cot B < 0 \) and \( \sec B > 0 \)

6. In which quadrant are cotangent and cosecant both negative?  
   a. I  
   b. II  
   c. III  
   d. IV

7. If \( \sin A \cot A > 0 \) and \( \sin A < 0 \), which must be true?  
   a. \( \cos A > 0 \)  
   b. \( \tan A > 0 \)  
   c. \( \sec A < 0 \)  
   d. \( \csc A > 0 \)
Everything you knew about sine, cosine, and tangent... let's apply it to secant, cosecant, cotangent.

1. Reciprocals of any value always have the same sign (+ or -) as the value of the original.

   This means:

   

   $\begin{array}{c|c|c|c|c|c|c|c|c|c}
   \theta & 0^\circ & 30^\circ & 45^\circ & 60^\circ & 90^\circ & 180^\circ & 270^\circ & 360^\circ \\
   \hline
   \text{Csc} \ \theta & & & & & & & & \\
   \text{Sec} \ \theta & & & & & & & & \\
   \text{Cot} \ \theta & & & & & & & & \\
   \end{array}$

   **Concept 4: Calculating exact values**

   Find the exact value of each expression.

   4. $\sec 300^\circ$
   5. $\csc 225^\circ$
   6. $\cot 270^\circ$
   7. $\cot 420^\circ$
   8. $\csc (-210^\circ)$
   9. $(\sec 150^\circ)(\cos 150^\circ)$
   10. $(\tan 300^\circ)(\cot 300^\circ)$
Summary

The Basic Trig Definitions

<table>
<thead>
<tr>
<th>Trig Function</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td>( \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} )</td>
</tr>
<tr>
<td>Cosine</td>
<td>( \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} )</td>
</tr>
<tr>
<td>Tangent</td>
<td>( \tan \theta = \frac{\text{opposite}}{\text{adjacent}} )</td>
</tr>
<tr>
<td>Cosecant</td>
<td>( \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} )</td>
</tr>
<tr>
<td>Secant</td>
<td>( \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} )</td>
</tr>
<tr>
<td>Cotangent</td>
<td>( \cot \theta = \frac{\text{adjacent}}{\text{opposite}} )</td>
</tr>
</tbody>
</table>

Given a right triangle with one of the angles named \( \theta \) with \( \theta \) in standard position, and the sides of the triangle relative to \( \theta \) named \( x, y, \) and \( r \) (picture on the right), we define the 6 trig functions to be:

<table>
<thead>
<tr>
<th>Trig Function</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td>( \sin \theta = \frac{y}{r} )</td>
</tr>
<tr>
<td>Cosine</td>
<td>( \cos \theta = \frac{x}{r} )</td>
</tr>
<tr>
<td>Tangent</td>
<td>( \tan \theta = \frac{y}{x} )</td>
</tr>
<tr>
<td>Cosecant</td>
<td>( \csc \theta = \frac{r}{y} )</td>
</tr>
<tr>
<td>Secant</td>
<td>( \sec \theta = \frac{r}{x} )</td>
</tr>
<tr>
<td>Cotangent</td>
<td>( \cot \theta = \frac{x}{y} )</td>
</tr>
</tbody>
</table>

The Pythagorean theorem ties these variable together: \( x^2 + y^2 = r^2 \)

Exit Ticket

1. In the diagram below of right triangle JTM, JT = 12, JM = 6, and \( m \angle JMT = 90 \).

![Diagram of right triangle JTM]

What is the value of cot \( J \)?

(1) \( \frac{\sqrt{3}}{3} \)  
(2) 2  
(3) \( \sqrt{3} \)  
(4) \( \frac{2\sqrt{3}}{3} \)

2. If \( \csc \theta = 2 \) and \( \sec \theta < 0 \), in what quadrant does \( \theta \) terminate?

(1) I  
(2) II  
(3) III  
(4) IV
Day 7 - The Reciprocal Trigonometric Functions HW

1) \( \sec \theta \)

2) \( \csc \theta \)

3) \( \sec \theta \)

4) \( \sec \theta \)

5) Find \( \tan \theta \) if \( \csc \theta = \frac{3}{2} \)

6) Find \( \cot \theta \) if \( \sin \theta = \frac{4}{5} \)

7) Find \( \csc \theta \) if \( \cot \theta = \frac{4}{3} \)

8) Find \( \csc \theta \) if \( \sec \theta = \frac{13}{12} \)

9) Find \( \sec \theta \) if \( \csc \theta = \frac{\sqrt{10}}{3} \)

10) Find \( \sin \theta \) if \( \cot \theta = \frac{4}{3} \)

Find the exact value of each trigonometric function.

11) \( \cot \theta \)

12) \( \cot \theta \)
13) $\sec \theta$

14) $\sec \theta$

15) $\sec \theta$

16) $\cot \theta$

17) $\cot \theta$

18) $\csc \theta$

19) $\cot \theta$

20) $\cot \theta$
21) \( \cot 135^\circ \)  
22) \( \sec 330^\circ \)  

23) \( \sec 0^\circ \)  
24) \( \cot 315^\circ \)  

Express each function as the function of a positive acute angle.

25) \( \csc -330^\circ \)  
26) \( \cot -135^\circ \)  

27) \( \csc -150^\circ \)  
28) \( \csc 330^\circ \)  

Find the value of each. Round your answers to the nearest ten-thousandth.

29) \( \sec 55^\circ \)  
30) \( \cot 70^\circ \)  

31) \( \cot 35^\circ \)  
32) \( \sec 25^\circ \)
For each point, DRAW the point on the coordinate plane, determine the hypotenuse by the pythagorean theorem, and find the sec, csc, and cot of the angle.

a) $P(12,9)$

b) $P(30,16)$

c) $P(1,2)$

d) $P(3,\sqrt{7})$

e) $P(-8,-6)$

f) $P(1,-3)$

g) $P(6,-\sqrt{13})$

h) $P(-\sqrt{2},-\sqrt{2})$
2. Given information about one trig function, find other trig functions:

   a) If \( \tan \theta = \frac{4}{3} \), \( \theta \) in quadrant I, find \( \cos \theta \) and \( \sin \theta \).  

   b) If \( \cos \theta = \frac{\sqrt{3}}{2} \), \( \theta \) in quadrant IV, find \( \sin \theta \) and \( \tan \theta \). 

   c) If \( \sin \theta = \frac{5}{8} \), \( \theta \) in quadrant II, find \( \sec \theta \) and \( \cot \theta \).  

   d) If \( \sec \theta = \frac{5}{2} \), \( \theta \) in quadrant III, find \( \sin \theta \) and \( \tan \theta \). 

   e) If \( \tan \theta = -5 \), \( \theta \) in quadrant IV, find \( \sin \theta \) and \( \sec \theta \).  

   f) If \( \cos \theta = \frac{\sqrt{2}}{3} \) and \( \sin \theta < 0 \), find \( \sin \theta \) and \( \tan \theta \). 

   g) If \( \sec \theta = \frac{6}{5} \), find \( \sin \theta \) and \( \tan \theta \). 

   h) If \( \tan \theta = \frac{4\sqrt{5}}{5} \), find \( \sin \theta \) and \( \cos \theta \).
3. In what quadrant is
   a) \( \sin \theta > 0 \) and \( \cos \theta < 0 \)    
   b) \( \csc \theta > 0 \) and \( \cot \theta < 0 \)
   
c) \( \sec \theta < 0 \) and \( \tan \theta < 0 \)    
   d) \( \csc \theta < 0 \) and \( \cos \theta < 0 \)

4. Find the value of the following (do not look at the chart – make a small picture and calculate the values)
   a) \( 5\sin 90^\circ - 7\cos 180^\circ \)    
   b) \( 4\sec 0^\circ + 7\csc 270^\circ \)
   
c) \( \sin^2 180^\circ + \cos^2 180^\circ \)    
   d) \( \left( 6\cot \frac{3\pi}{2} + 3\sec \pi \right)^3 \)
   
e) \( \cos 0^\circ \sin 270^\circ - \cos 270^\circ \sin 0^\circ \)    
   f) \( \left( \sin 270^\circ - \sec 0^\circ \right) \left( \sin 270^\circ + \sec 0^\circ \right) \)

6. Find the value of the following (do not look at the chart – make a small picture and calculate the values)
   a) \( 6\sin 30^\circ - 4\cos 150^\circ \)    
   b) \( 8\sin 60^\circ - 4\sin 300^\circ \)
   
c) \( (4\tan 120^\circ)(8\cos 225^\circ) \)    
   d) \( 6\sin 315^\circ + 8\tan 135^\circ \)
   
e) \( \frac{8\csc 30^\circ}{\cot 330^\circ} \)    
   f) \( -2\cos 225^\circ - 4\cot 315^\circ + 3 \)
   
g) \( \sin^2 225^\circ - \cos^2 225^\circ \)    
   h) \( \cos^2 630^\circ - \csc^2 (-30)^\circ \)
Day 7 – HW Answers

Answers to Algebra2/Trig: The Reciprocal Trig Functions (Amsco 9-5) (ID: 1)

1) \( \frac{5}{4} \)  
2) \( \frac{7}{2} \)  
3) \( \frac{5}{3} \)  
4) \( \frac{5}{4} \)  
5) \( \frac{2\sqrt{5}}{5} \)  
6) \( \frac{3}{4} \)  
7) \( \frac{5}{3} \)  
8) \( \frac{13}{5} \)  
9) \( \sqrt{10} \)  
10) \( \frac{3}{5} \)  
11) \( -\frac{\sqrt{3}}{3} \)  
12) \( -1 \)  
13) \( -\frac{2\sqrt{3}}{3} \)  
14) \( \sqrt{2} \)  
15) \( -\sqrt{2} \)  
16) \( 0 \)  
17) \( \sqrt{3} \)  
18) \( \frac{2\sqrt{3}}{3} \)  
19) \( -\sqrt{3} \)  
20) \( -\sqrt{3} \)  
21) \( -1 \)  
22) \( \frac{2\sqrt{3}}{3} \)  
23) \( 1 \)  
24) \( -1 \)  
25) \( 2 \)  
26) \( 1 \)  
27) \( -2 \)  
28) \( -2 \)  
29) \( 1.7434 \)  
30) \( 0.3640 \)  
31) \( 1.4281 \)  
32) \( 1.1034 \)
1) Which angle is coterminal with an angle of 125°?

[C] -235°  [D] 235°

2) If \( \theta \) is an angle in standard position and its terminal side passes through the point \( \left( \frac{1}{2}, \frac{-\sqrt{3}}{2} \right) \) on a unit circle, a possible value of \( \theta \) is

[A] 150°  [B] 30°  [C] 60°  [D] 120°

3) In the unit circle shown in the accompanying diagram, what are the coordinates of \((x, y)\)?

\[
\begin{align*}
(x, y) & \quad \text{(30°)} \\
\end{align*}
\]

[A] \( \left( -\frac{\sqrt{3}}{2}, -0.5 \right) \)  [B] \( (-0.5, -\frac{\sqrt{3}}{2}) \)
[C] \( \left( -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right) \)  [D] \( (-30, -210) \)

4) If \( x \) is a positive acute angle and \( \cos x = \frac{\sqrt{3}}{4} \), what is the exact value of \( \sin x \)?

[A] \( \frac{\sqrt{3}}{5} \)  [B] \( \frac{3}{5} \)  [C] \( \frac{\sqrt{13}}{4} \)  [D] \( \frac{4}{5} \)
5) Two straight roads intersect at an angle whose measure is $125^\circ$. Which expression is equivalent to the cosine of this angle?

[A] $-\cos 35^\circ$     [B] $\cos 55^\circ$

[C] $\cos 35^\circ$     [D] $-\cos 55^\circ$

6) If the tangent of an angle is negative and its secant is positive, in which quadrant does the angle terminate?


7) If $\sin x = \frac{1}{a}$, $a \neq 0$, which statement must be true?

[A] $\csc x = a$     [B] $\csc x = a$

[C] $\sec x = -\frac{1}{a}$     [D] $\csc x = -\frac{1}{a}$

8) The expression $\frac{\sec \theta}{\csc \theta}$ is equivalent to

[A] $\frac{\sin \theta}{\cos \theta}$     [B] $\cos \theta$

[C] $\sin \theta$     [D] $\frac{\cos \theta}{\sin \theta}$

9) What is the length of the altitude of an equilateral triangle whose side has a length of 8?

[A] 4   [B] 32   [C] $4\sqrt{3}$   [D] $4\sqrt{2}$

10) If the sine of an angle is $\frac{3}{5}$ and the angle is not in Quadrant I, what is the value of the cosine of the angle?
11) In the accompanying diagram, point \( P(0.6, -0.8) \) is on unit circle \( O \). What is the value of \( \theta \), to the nearest degree?

![Diagram of unit circle with point P(0.6, -0.8)]

12) If \( \theta \) is an angle in standard position and its terminal side passes through the point \((-3, 2)\), find the exact value of \( \csc \theta \).

13) The accompanying diagram shows unit circle \( O \), with radius \( OB = 1 \).

Which line segment has a length equivalent to \( \cos \theta \)?

[A] \( \overline{AB} \)  [B] \( \overline{OC} \)  [C] \( \overline{OA} \)  [D] \( \overline{CD} \)
14) Find the exact value.
   a) \( \sin 30^\circ \)  
   b) \( \cos 135^\circ \)  
   c) \( \cot 60^\circ \)  
   d) \( \sec 45^\circ \)  
   e) \( \cos 330^\circ \)  
   f) \( \tan 210^\circ \)  

15) The terminal side of \(<ROP\) in standard position intersects the unit circle at \( P \). If \( m<ROP \) is \( \theta \), find:
   a) the quadrant of \(<ROP\)  
   b) \( \sin \theta \)  
   c) \( \cos \theta \)  
   d) \( \tan \theta \)  
   e) \( \csc \theta \)  
   f) \( \sec \theta \)  
   g) \( \cot \theta \)

   \( P \left( -\frac{2}{3}, -\frac{\sqrt{5}}{3} \right) \)

16) The given point is on the terminal side of angle \( \theta \) in standard position. Find:
   a) the quadrant of the angle, 
   b) \( \sin \theta \), c) \( \cos \theta \), d) \( \tan \theta \), e) \( \csc \theta \), f) \( \sec \theta \), g) \( \cot \theta \).

   \( P(9, -13) \)
17) Construct the table for your special angle trig values.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Sin</th>
<th>Cos</th>
<th>Tan</th>
</tr>
</thead>
</table>

18) Find the exact value of each expression
   a. \( \csc 120^\circ \)   b. \( \tan 405^\circ \)   c. \( \cos 135^\circ \)   d. \( \tan (-120^\circ) \)

19) Express each of the following as a function of an acute angle
   a. \( \cos 190^\circ \)   b. \( \tan 305^\circ \)   c. \( \sec 92^\circ \)   d. \( \sin (-165^\circ) \)

20) Find each function value to four decimal places
   a. \( \sin 43^\circ 20' \)   b. \( \cos 77^\circ 50' \)   c. \( \tan 39^\circ 46' \)

21) Find the measure of \( \theta \) to the nearest minute
   a. \( \sin \theta = .9390 \)   b. \( \cos \theta = .1113 \)   c. \( \tan \theta = 1.1868 \)

22) If \( \tan \theta = -\frac{3}{4} \) and \( \sin \theta > 0 \), find:
   a. \( \sec \theta \)   b. \( \cos \theta \)   c. \( \sin \theta \)   d. \( \csc \theta \)   e. \( \cot \theta \)

23) Write each expression in terms of \( \sin \theta \), \( \cos \theta \), or both, in simplest form.
   a. \( \frac{\sec \theta}{\tan \theta} \)   b. \( \csc \theta \sin^2 \theta \)